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Monetary Aggregation and the Demand for Assets

In this paper we consider and illustrate a solution to the inter-related problems of monetary aggregation and estimation of money demand. The problem with the definition of money is that the relative prices of the monetary components fluctuate over time, rendering simple-sum aggregates inefficient. We apply Revealed Preference tests to the U.S. monthly data to determine admissible and separable components. These components are then aggregated using the Divisia technique. To deal with the problem of money demand, the dynamic Fourier expenditure system is used to provide estimates of the elasticities of substitution. These, while showing general substitution among the liquid assets studied are quite variable over time. This finding underscores the inefficiency of both simple-sum aggregation and single-equation, log-linear money-demand estimation.

IN RECENT YEARS the behavior of the monetary aggregates defined by the Federal Reserve has been sufficiently erratic to provoke considerable concern about their usefulness in a well-designed monetary policy. Since the "missing money" episodes of the 1970s, the traditional measures of M1 and M2 have provided inconsistent and sometimes surprising results; similarly, velocity measures based on the same concepts have often gone way off track, by almost any standard. Doubts about the stability of the demands for these entities also surface repeatedly, and certainly cannot be discarded, but the most telling criticism concentrates on the components of these aggregates themselves.

The basic problem with the aggregates is that central bank's simple-sum method of aggregation will not produce theoretically satisfactory definitions of money if the relative prices of the monetary components fluctuate over time. The problem is an incorrect accounting for substitution effects resulting in a set of monetary aggregates that do not accurately measure the actual quantities of the monetary services that optimizing economic agents select (in the aggregate). As illustrated in Barnett, Fisher, and Serletis (1992), the fluctuations in the underlying prices (user costs) of the components of the monetary aggregates are too large to ignore.

With respect to estimates of the demand for money, we believe that a demand-

Adrian R. Fleissig acknowledges the support of the SLU Beaumont Faculty Development Fund.

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Journal of Money, Credit, and Banking, Vol. 29, No. 4 (November 1997, Part 1)

systems approach would be effective. What we have in mind has been done before,but has failed to win general acceptance.[1] We believe that a major problem in winning acceptance for the demand-systems approach is that the existing literature has paid little attention to the dynamic context in which policy operates; implementing an effective way to do this is the main contribution of this study. We also feel that by using quarterly instead of monthly data, the existing studies fail to use a temporal aggregation that is anywhere near the most useful policy framework; this is no longer than monthly, if the regularly scheduled meetings of the Federal Open Market Committee are any guide. Finally, we argue that either Morishima or McFadden elasticities give more insight into the actual substitution between monetary assets than the traditional Allen-Uzawa measure typically used in such studies.

We tackle the problem in three stages. First, we apply the revealed preference approach (Varian 1982, 1983; Swofford and Whitney 1987, 1988) to determine the *components* of optimal monetary aggregate aggregates; this exercise produces a collection of assets that differs considerably from the set employed by the Federal Reserve. Second, we employ an "ideal" index number—the Divisia—to perform the aggregation of the components identified in the first stage. We will explain why below. Third, employing these aggregates we estimate a system of demand equations and calculate the substitution elasticities. These vary considerably over the data period (1960–1993) especially at business cycle turning points. We argue that this variation interferes with the successful use of simple-sum aggregates and traditional log-linear money-demand functions.

1. MONETARY AGGREGATION

Most of the studies of money in the literature employ monetary aggregates that are simple sums of their components (for example, M1A equals currency plus deposits). The simple-sum aggregation might serve policymakers well when interest rate fluctuations are relatively mild but not when the interest rates on the monetary components fluctuate significantly, which is often the case, especially in recent years. Any aggregation procedure (for example, simple-sum, Laspeyres, and Divisia indices) should internalize the substitution effects (at constant utility) that arise from relative price changes of the component series. This result will hold for simple-sum aggregation only if all the assets added together are perfect substitutes. For example, the simple-sum M2 aggregate would require currency to be a perfect substitute for money market mutual funds; this is highly unlikely.

What will work in practice is to employ an aggregation procedure that is directly linked to optimization theory. The optimization theory is that applicable to the aggregate consumer. The general consumer optimization framework has the agent in

1. See Ewis and Fisher (1984, 1985), Barnett and Yue (1988), Swofford and Whitney (1987, 1988), Belongia and Chalfant (1989), Barnett and Spindt (1982), Serletis (1988, 1991), Barnett, Fisher and Serletis (1992), Fisher (1992). Chrystal and Drake (1994), Fisher and Fleissig (1994). The Barnett, Fisher, and Serletis study surveys a good deal of this literature.

each period undertaking labor supply decisions and determining how to allocate his actual income over consumption goods (c) and monetary assets (m). Following Dusansky (1989), Samuelson and Sato (1984), and Barnett, Fisher, and Serletis (1992), if service flows from these three entities enter as arguments in the agent's utility function, then

$$u = U(\mathbf{c}, L, \mathbf{m}) \tag{1}$$

where **c** and **m** are vectors of services from consumption goods and monetary assets and L is leisure.[2] The utility function is maximized subject to the full income constraint (y)

$$y = \mathbf{p}'\mathbf{c} + \boldsymbol{\pi}'\mathbf{m} + wL \tag{2}$$

where p is a vector of prices of c, π is a vector of monetary user costs, and w is the shadow price of leisure. The user cost for the monetary assets in this problem, as defined by Donovan (1978) and derived by Barnett (1978) in a constrained consumer optimization framework, is

$$\pi_i = p^* \frac{R - r_i}{1 + R} \tag{3}$$

where p^* is the true cost-of-living index, R is the rate of return on a bench mark asset, and r_i is the own rate of return from the ith monetary asset. The user cost for consumer goods is that derived by Diewert (1974b).

What is required in order to construct consistent aggregates is a procedure that is theoretically connected to the problem just stated (this is called a "superlative index" in the literature) and hence can deal with the less than perfect substitutability that one finds in practice. The Divisia Index is one such. In fact, the Divisia is actually derived from the consumer optimization problem being a simple transformation of the first-order conditions; it has other desirable properties as (see Barnett, Fisher and Serletis 1992). Indeed, if the real world actually presents us with perfect substitutability, the Divisia will give the same answer as the simple sum. This establishes its superiority, for practical purposes.

Having a satisfactory index such as the Divisia does not, however, determine what set of assets to consider or how to group the assets into subsets of the data for efficient estimation. A procedure that is available for both purposes is the linear NONPAR program of Varian (1982, 1983) that is based directly on the Generalized Axiom of Revealed Preference (GARP). The procedure is applied in two stages. One first tests the entire proposed data set (the components of the official monetary aggregates and the subcategories of consumption) for its consistency with utility-

2. Inserting money in the utility function is equivalent to putting money in the budget constraint for a broad class of utility functions (see Arrow and Hahn 1971, Feenstra 1986, and Barnett, Fisher and Serletis 1992).

maximizing behavior (that is, with GARP). If GARP is satisfied, then one can search among the specific assets for specific *separable* groupings of the data. If any groupings can be established—that is, if weak separability holds among the subgroups of the overall collection—then these groups are available for aggregation, within each group.[3] A satisfactory procedure for aggregation is by means of the Divisia Index. These totals can then be analyzed further, as we do, utilizing a demand-system approach.

1.1 Data

The Federal Reserve Bank of St. Louis provides monthly data on the quantities of twenty-four monetary assets during 1960:01–1993:05 along with the corresponding "own" rates of return that are used in calculating the user costs.[4] Time series for monthly durables, nondurables, services, consumer price index, wages and disbursements, labor hours, and total civilian population are from the Citibank data base. All series are converted into 1987 real per capita term using the CPI and total civilian population.[5] The construction of the stock of durable goods follows the procedure used by Campbell and Mankiw (1990) as modified by Fleissig, Hall, and Seater (1994).[6] We also have to construct user costs for these entities; we employ the procedures of Barnett (1978) and Diewert (1974b) that were described earlier.

1.2 Revealed Preference Results

As discussed above, an appropriate procedure for detecting appropriate aggregation strategies is the NONPAR algorithm of Varian that is based on the General Axiom of Revealed Preference. The NONPAR procedure is nonstochastic and a single violation produces a rejection. Violations are certainly likely with monthly data. One problem that arises is that any time the number of assets or commodities in the data set changes, the GARP tests cannot be performed over the entire period because of missing values.[7] Our original sample of monthly data is from

3. According to the Leontief-Sono definition of separability, weak separability requires that the marginal rates of substitution between any two commodities in the proposed grouping be independent of changes in relative prices outside the group.

4. The data are from Thornton and Yue (1992) and are constructed using the same techniques as Farr and Johnson (1985).

5. By casting the data in per capita form, we are, in effect, invoking the "representative agent" framework.

6. The standard equation linking expenditure on durables (dur_t) and the stock of durables (K_t) is $K_t = dur_t + (1 - \delta)K_{t-1}$, where δ = depreciation rate of the stock of durables. The *Survey of Current Business* only publishes data for the annual stock of consumer durables. The estimate for the initial monthly stock of durables (1960:01) is the year-end 1959 annual stock. A grid search is then performed to find a depreciation rate (δ) that produces the January 1991 monthly durable stock equal to the annual stock for 1990; that is, the 1990 annual stock can be considered to be the initial value for the stock in January 1991. The interpolated monthly depreciation rate is .01625 (19.4 percent per year) which is very close to the 20 percent depreciation rate used by Christensen and Jorgenson (1973), whereas Diewert (1974) and Swofford and Whitney (1987, 1988) use 10 percent per year. The monthly depreciation rate may seem high but is compensating for the fact that the annual stock of durables includes discards; that is, $K_t = dur_t + (1 - \delta)K_{t-1} - discards$.

7. One such was the introduction of money market mutual funds in 1973:11.

1960:01–93:05; it is broken down into twelve subperiods due to financial innovations. Using the complete set of assets produced by the Federal Reserve and the consumption goods, we find that for five of these twelve periods there are one or more violations of GARP. These five subperiods cover much of the data (1963:01–1969:08, 1970:02–1973:10, 1977:02–1982:11, 1983:01–1986:03, and 1986:04–1991:08).

There are no statistical procedures available to test to see if the number of violations of GARP is statistically significant.[8] As a partial remedy, Chalfant and Alston (1988) and Varian (1990) suggest procedures that provide some insight into the potential damage that might be inflicted by revealed preference violations. Chalfant and Alston look to see if the data are consistent with the Weak Axiom of Revealed Preference (WARP) which is itself a necessary condition for a data set to satisfy GARP.[9]

In Table 1 we report the WARP violations for the five periods for which there were also GARP violations. In the table, the longest period that produces WARP violations occurs between observations 1981:5 and 1982:11. Here the percentage of successful observations is .997510; this seems a small percentage of error. In fact, for all of the subperiods for which there were GARP violations, the percentage of WARP errors is strikingly small.

As already noted, there is a second procedure, developed by Varian (1990) that is available to evaluate how serious the violations of GARP might be. Basically, Varian provides a goodness-of-fit test that gives a value of unity if exact optimization occurs. Values less than unity imply less than exact optimization. We would not expect exact optimization, of course, and we cannot say how far from perfect optimization is too far, but the goodness-of-fit indices displayed in the last column of Table 1 indicate that the observed choice behavior was at the worst 96.1 percent as efficient as exact maximization behavior on our data. Just as with the WARP procedure, the goodness-of-fit index provides reason for optimism about the further use of the monetary quantities and their prices.

If GARP is not satisfied, then weak separability cannot be demonstrated (by the NONPAR procedure) and aggregation cannot proceed; thus entities such as M1, M2, etc. cannot be shown to be consistent with rational consumer behavior. A way to proceed is to break the data into subperiods that actually do satisfy GARP so that one can then test for weak separability. We were able to establish the existence of twelve such subperiods using the points of failure of GARP as breakpoints; these periods were further divided to produce a set of twenty-one subsubperiods, each of

8. The test procedure of Varian (1985) and Swofford and Whitney (1994) is only applicable for small data sets and cannot be applied to the data set used in this paper.

9. They define a matrix Φ with elements, $\phi_{ij} = P_i'X_j$, representing the cost of quantity vector X_j at P_i prices. For WARP, the consumption vector X_i is revealed preferred to X_j, written as X_iRX_j, if $\phi_{ii} > \phi_{ij}$. If it is also true, given the X_i was affordable at prices P_j, that $\phi_{jj} > \phi_{ji}$ *then* X_jRX_i; this is a violation of WARP. Therefore, when the ratio $\phi_{ij}/\phi_{ii} < 1$ it implies that X_iRX_j. Chalfant and Alston (1988) use the ratios ϕ_{ij}/ϕ_{ii} and ϕ_{ji}/ϕ_{jj} to judge the importance of WARP violations. For example, a ratio $\phi_{ij}/\phi_{ii} = .99$ indicates that the commodity vector is period *j*, X_j, was just 1 percent cheaper than the commodity vector that was actually consumed.

TABLE 1
TESTS OF VIOLATIONS OF REVEALED PREFERENCE

Sample	Year *i*	Year *j*	WARP (*i*, *j*)	WARP (*j*, *i*)	Goodness-of-Fit Index
63:01–69:08	63:8	69:5	.997582	.999351	.993
	64:5	66:10	.999857	.999934	
	64:9	69:6	.998582	.998906	
	64:9	69:7	.998145	.999845	
	68:1	69:8	.999961	.999924	
70:02–73:10	70:07	70:08	.999998	.999989	.999
77:02–82:11	80:5	80:11	.999876	.999096	.987
	81:5	81:10	.999564	.999609	
	81:5	82:11	.997510	.997773	
	81:8	82:4	.999898	.999845	
	81:9	82:5	.999733	.999179	
83:01–86:03	84:4	85:8	.999789	.999790	.973
	84:6	85:7	.999998	.998710	
	84:11	85:6	.999389	.999913	
86:04–91:08	86:11	91:04	.999899	.999998	.961
	88:01	88:05	.999998	.999990	

which actually passes GARP over the entire sample of all the assets.[10] These periods were then tested to see if weakly separable groupings could be established for the traditional Federal Reserve definitions of M1A, M1, M2, M3, and L. Of these aggregates, M1 passed separability tests for only 51.4 percent of the months and the other Federal Reserve definitions did even worse.[11]

Considering that the Fed's groupings of assets fail separability tests over many subperiods, are there some alternative arrangement of assets that do not? It turns out, after much testing, that the following groups always pass the necessary condition for weak separability over the entire set of subperiods of the data.[12]

CUR, CDD, OCD, SNOWC, SNOWT	A1
SDCB, SDSL	A2
STDCB, STDTH	A3
DUR, NOND, SER, LEIS	A4

Here CUR is currency, CDD are demand deposits for households, OCD are other checkable deposits, SNOWC and SNOWT are super NOW accounts at commercial banks and thrifts, SDCB and SDSL are savings deposits at commercial banks and S&Ls, STDCB and STDTH are small time deposits at commercial banks and thrifts, and DUR, NOND, SER, and LEIS are the obvious subcomponents of aggregate consumption. Group A1 is M1 excluding business demand deposits and groups

10. That is, the result of taking the entire set of data, but testing for GARP violations over each of the twenty-one sub-subperiods separately, produced no violations of GARP for the entire set of data provided by the Federal Reserve.

11. The percent of months consistent with weak separability for the remaining aggregates are M1A = 36.2%, M2 = 34.7%, M3 = 40.1%, and L = 37.9%.

12. Swofford and Whitney (1987, 1988) obtain similar results using quarterly data, without the use of subperiods.

A2 and A3 contain assets from M2 that are typically held by individuals. No groupings including money market funds pass the weak separability tests over all the subsamples.[13]

We think that the exclusion of business items from these separable groupings makes economic sense. Clearly, this points up a major reason why the traditional measures of money perform so badly in GARP tests. These measures include items that are held for different reasons by different economic agents (businesses and consumers) who may react differently to the same (or even to different) variables. This is an additional objection to the simple sum than the argument already advanced about the failure to deal adequately with substitution effects.

Returning, then, to the empirical work, the next step, having established separable groupings of the data, is to construct aggregates of these subcategories. What we do, to attempt to preserve the economic characteristics of these sets of data up to a third-order remainder term, is to construct Divisia index numbers from the individual quantities and their associated user costs; these are designated as A1, . . . , A4 in the list given above.[14] We then employ the demand-systems approach to provide estimates of the substitution elasticities among A1, . . . ,A4, allowing for dynamic behavior. We do this to show both the extent of the variability of substitution parameters over time and the low values of the substitution elasticities.

2. ECONOMETRIC CONSIDERATIONS

In order to obtain consistent estimates of the elasticities of substitution among financial assets, we estimate a system of demand equations derived from the indirect utility function. In effect, an accurate approximation to the indirect utility function is required and the optimal strategy is to employ a flexible functional form to provide this approximation. Because the elasticities are functions of the derivatives of the indirect utility function, the flexible form should be able to provide a precise global approximation to the true but unknown indirect utility function, including its derivatives.

Individuals generally react to past decisions and face various constraints that imply less than full adjustment of consumption plans and asset holdings during each period;[15] this is certainly likely to be a factor on the monthly financial data that we are examining in this study. A direct approach to modeling the dynamics is to allow past behavior to affect current decisions directly through the utility function. The only dynamic flexible function in the literature to date possessing such qualities is the dynamic Fourier flexible form developed by Fisher and Fleissig (1994) from

13. Swofford and Whitney (1987, 1988) on quarterly and annual data also found that money market funds were excluded by the GARP procedure. The Belongia-Chalfant study (1989) does include these funds successfully, but they use nominal rather than real entities. We therefore think that our finding is consistent with this small literature, but is, nevertheless, puzzling.

14. Following Farr and Johnson (1985) and Thornton and Yue (1992) the Fisher Ideal index must be used instead of the Divisia over the periods where there is missing data, that is, were zeros appear.

15. See, for example, Swofford and Whitney (1994), who find incomplete adjustment on U.S. monetary data.

Gallant's (1981) static version. In fact, the dynamic Fourier can provide an extremely accurate approximation to the true indirect utility function and its derivatives even if that function is highly nonlinear. This property of the Fourier is very useful for analyzing money demand considering that there are violations of GARP on the data set and that there is considerable volatility of interest rates during the period studied.

The dynamic Fourier flexible form of Fisher and Fleissig (1994) is

$$G_k^d(\mathbf{z}, \theta) = u_o + \mathbf{b}'\mathbf{z} + \frac{1}{2}\mathbf{z}'\mathbf{C}\mathbf{z} + \sum_{\alpha=1}^{A} \left(u_{o\alpha} + 2\sum_{j=1}^{J} [u_{j\alpha}\cos(j\mathbf{k}_\alpha'\mathbf{z}) - w_{j\alpha}\sin(j\mathbf{k}_\alpha'\mathbf{z})] \right) \qquad (4)$$

in which

$$C = -\sum_{\alpha=1}^{A} u_{0\alpha}\mathbf{k}_\alpha\mathbf{k}_\alpha'$$

and u_0, b, u_{01}, u_{02}, . . ., w_{11}, w_{12}, . . .are the parameters to be estimated and z is a vector of the current and lagged expenditure-normalized user costs of the particular assets involved in the exercise. The dynamic shares are obtained by applying Roy's identity to equation (4), where $i = 1, \ldots, n$.

The share equations can be more compactly expressed as

$$y_{it} = \frac{z_{it}b_i - \sum_{\alpha=1}^{A}(u_{0\alpha}\mathbf{z}'\mathbf{k}_\alpha + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(j\mathbf{k}_\alpha'\mathbf{z}) + w_{j\alpha}\cos(j\mathbf{k}_\alpha'\mathbf{z})])k_{i\alpha}z_{it}}{\sum_{i=1}^{n} b_i z_{it} - \sum_{\alpha=1}^{A}(u_{0\alpha}\mathbf{z}'\mathbf{k}_\alpha + 2\sum_{j=1}^{J} j[u_{j\alpha}\sin(j\mathbf{k}_\alpha'\mathbf{z}) + w_{j\alpha}\cos(j\mathbf{k}_\alpha'\mathbf{z})])\mathbf{k}_\alpha'\mathbf{z}} \,. \qquad (5)$$

$$y_t = f(\mathbf{z_t}, \theta) \,. \qquad (6)$$

This system is what is estimated in the following section.

3. EMPIRICAL RESULTS

The money demand equations, with the across-equation restrictions, are estimated using International TSP's seemingly unrelated regression procedure. As with vector autoregressions and other time series methods frequently used in the literature, there can be many parameters to estimate; not all need be significant. This is

TABLE 2

DYNAMIC FOURIER FLEXIBLE FUNCTIONAL FORM AT SIX LAGS

	MSE	RMSE	*R* Square	*Q*-Statistic
Share 1	0.02026209	0.0000506552	0.9896237	2.50
Share 2	0.02501935	0.0000625484	0.9890560	3.07
Share 3	0.08125714	0.0002031428	0.9944605	2.50

Parameter	Estimate	Standard Error	*t*-statistic
*b*1	−.07999753	.01076245	−7.4330212
*b*2	.24016276	.09327136	2.5748821
*b*3	.98311008	.30002088	3.2768055
*u*01	.00059812	.00027239	2.1957886
*u*02	−.00723985	.00158179	−4.5769986
*u*03	−.00258979	.00082017	−3.1576192
*u*04	.02363551	.00675297	3.5000145
*u*11	.00033245	.00026070	1.2751919
*u*12	−.00064677	.00071904	−0.8994939
*u*13	.00082894	.00052399	1.5819827
*u*14	.00265162	.00301698	0.8788935
*w*11	.00007110	.00025974	0.2737314
*w*12	.00049743	.00071086	0.6997599
*w*13	.00081610	.00051384	1.5899792
*w*14	−.00654340	.00331282	−1.9751782

not a drawback, as long as the fit is good and many of the estimated parameters are actually significant. The share equations given in equations (5) and (6) are estimated with an error term appended. It turns out that there is autocorrelation in this dynamic system. Berndt and Savin (1975) show that a satisfactory way of dealing with the problem is to restrict the autocorrelation parameter to be the same for each of the equations in the system. If this were not the case, the results would differ depending on which equation is dropped from the system during estimation.[16]

As noted, the demand system in equations (5) and (6) is the one actually estimated. It is often suggested that it takes economic agents from six to eighteen months to adjust their portfolios of liquid assets to changes in interest rates. To accommodate for varying lengths of adjustment to equilibrium, we estimated the demands for monetary assets by allowing for six, twelve, and eighteen months of adjustment.

We begin our discussion by examining the Fourier estimates allowing for six months of adjustment, as shown in Table 2. It is quite clear that each share equations provide an accurate approximation to the data in terms of the root mean square error (RMSE), the R-square, and the values of the Q-statistic. Indeed, the Box-Pierce test for autocorrelation of the residuals for each share indicate white noise at the 5 percent level. In addition, a Wald test shows that the static Fourier, which is a special case of the dynamic, is rejected at the 1 percent level. In what follows, the parameter estimates from the Fourier are used to calculate the elasticities of substitution.

16. The traditional method is to omit one of the equations when the system is estimated. In addition, because the parameters are homogeneous of degree zero, b_4 is normalized to -1 (see Gallant 1981).

3.1 Elasticities of Substitution: Theoretical Considerations

In theoretical models that contain only two variables, the elasticity of substitution relationship between these variables is unambiguous; the variables must be substitutes. When there are more than two variables the elasticity of substitution relationship between x_i and x_j becomes more complex and depends, for example, on which direction one takes in moving toward the point(s) of approximation. Following Mundlak (1986), we consider three well-known possibilities.

The one-price-one-factor elasticity, such as the traditional Allen-Uzawa elasticity of substitution, is defined as

$$\frac{\hat{x}_i}{\hat{v}_j} = \frac{d\ln x_i}{d\ln v_j}$$

where v_i is the user cost of asset x_i. This measure, as Chambers (1988) and Blackorby and Russell (1989) argue, is of limited use because it measures only how an asset demand changes in response to a change in the user cost of an alternative asset (j). The Allen-Uzawa measure is symmetric by construction and fails to give information about how relative asset demands x_i/x_j change in response to a change in the user cost of asset i. Such a symmetric response is not a requirement of demand theory, since the result depends on the gradient, as already noted.

A measure that takes the possibility of nonsymmetry into account is the two-factor-one-price elasticity of substitution of Morishima. This can be expressed as

$$\frac{\hat{x} - \hat{x}_j}{\hat{v}_j},$$

an expression that measures how the asset ratio x_i/x_j changes as the user cost v_j changes. For a multiasset framework (we have four), this expression yields different values for changes in v_i and v_j and, therefore, captures the lack of symmetry that seems possible in such calculations. The actual method of calculating the Morishima elasticities is the following (see Blackorby and Russell 1989).

$$M_{ij} = \epsilon_{ji} - \epsilon_{ii} \qquad \text{where} \qquad \epsilon_{ji} = \frac{v_i C_{ij}}{C_i}.$$

Here c_i is the derivative of the cost function with respect to the ith input and c_{ij} is the second derivative of c with respect to input j.[17]

A third measure of the elasticity of substitution is the two-asset-two-price elasticity

$$\frac{\hat{x}_i - \hat{x}_j}{\hat{v}_j - \hat{v}_i},$$

17. The Morishima cross-elasticity, M_{ij}, is asymmetric, $M_{ij} \neq M_{ji}$, unless the utility function is CES.

which shows the percentage adjustment in asset ratios associated with changes in user-cost ratios. These elasticities are again symmetric and do not distinguish specific changes in the user cost of assets for i and j. An example is the McFadden (1986) shadow elasticity (MS_{ij}) which is a share-weighted average of the Morishima elasticity and is expressed as follows (Chambers 1988, p. 97).

$$MS_{ij} = \frac{S_i}{S_i + S_j} M_{ij} + \frac{S_j}{S_i + S_j} M_{ji} .$$

In what follows we will show examples of each, although our preference is for the Morishima elasticity principally because there does seem to be a lack of symmetry for our data set.

The dynamic Fourier model provides precise estimates of the elasticities of substitution at each data point. From our point of view, the most interesting numbers are the cross-elasticities of substitution, since these can be used to measure closeness of substitution (or even complementarity) among monetary assets. These numbers will change over time, as the user costs for the various assets alter. The reason, clearly, is that we are sliding around the aggregate asset-holder's utility (hyper-) surface. For large changes in user costs such as those associated with recent business cycles, there could be large changes in the estimated elasticities. Institutional changes that alter the own rates of return on these assets—and hence their user costs—are also likely causes of changing elasticities.

3.2 Empirical Comparisons

We do not have the space to undertake a full review of the many ways the substitution elasticities could be compared, but in our work certain relationships appear to stand out. We begin with a set of elasticities that go with Table 2, which reported parameter estimates for the dynamic Fourier system allowing for six months of adjustment. The results are for Morishima elasticities between A1 and A2, A1 and A3, and A1 and A4, with the user cost of A1 varying and are displayed in Figure 1. For these results, it is apparent that the financial assets are always substitutes (positive numbers for the elasticity of substitution) but that the values are not very large, rising only to around .6 in the period.

The elasticity calculated between A1 and the composite consumption commodity (A4) is generally quite low and has a pattern not at all like the elasticities for the consumption goods. With respect to the financial asset aggregates (A1, A2, and A3), it seems that there is considerable variation around the recessions in 1970, 1975–5, 1980–2, and 1991. This is shown equally by the two measures.

One also notices that after 1979—after, that is to say, double-digit inflation, monetary decontrol, and the disinflation of the early 1980s—there is more variability in the estimated elasticities among the financial assets. These results are neither unexpected nor improbable. If the surfaces that we are approximating are highly nonlinear and if user costs vary considerably (see below), then the elasticities will show this behavior. It is transparent, if these results are correct, that procedures that re-

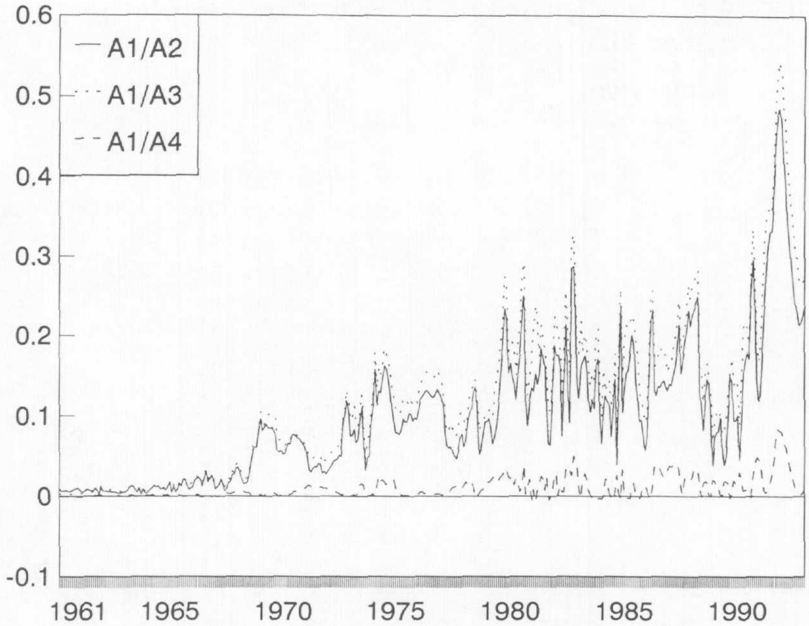

FIG. 1. Short-Run Elasticities of Substitution, Six Lags, August 1960–May 1993

quire very close substitution or constant elasticities of substitution will not capture the behavior that is exhibited here. We believe this exhibits the fundamental difficulty with the traditional measures of money.

Economic theory suggests that the absolute values of short-run elasticities are less than long, since longer periods of time permit more adjustment to occur. To illustrate this, Figure 2 presents a comparison between the short-run and long-run elasticities for A1 and A3 (varying the user cost of A1) at eighteen lags. This is as long an adjustment period as we estimated. Here the same patterns as observed in Figure 1, around cyclical turning points, are evident. Evident also is the generally larger value for the long-run elasticity than for the short-run. Recall that A1 is cash assets and A3 is, approximately, time deposits, both held by consumers.

The Morishima elasticities are not constrained to be symmetric. To illustrate both that there is a difference and that it does not change any conclusions fundamentally, we exhibit the elasticities between A2 and A3, the savings deposits and time deposits of households. The elasticity A2/A3 is calculated varying the user cost of savings deposits (A2) and that for A3/A2 is calculated using the user cost of time deposits (A3). Figure 3 shows the results. There clearly are differences in the elasticities estimated, but both the time patterns and the general finding of substitution between the two assets is confirmed.

In order to see whether the Morishima and McFadden methods of estimating elasticities produce different results, we have prepared yet another comparison, concen-

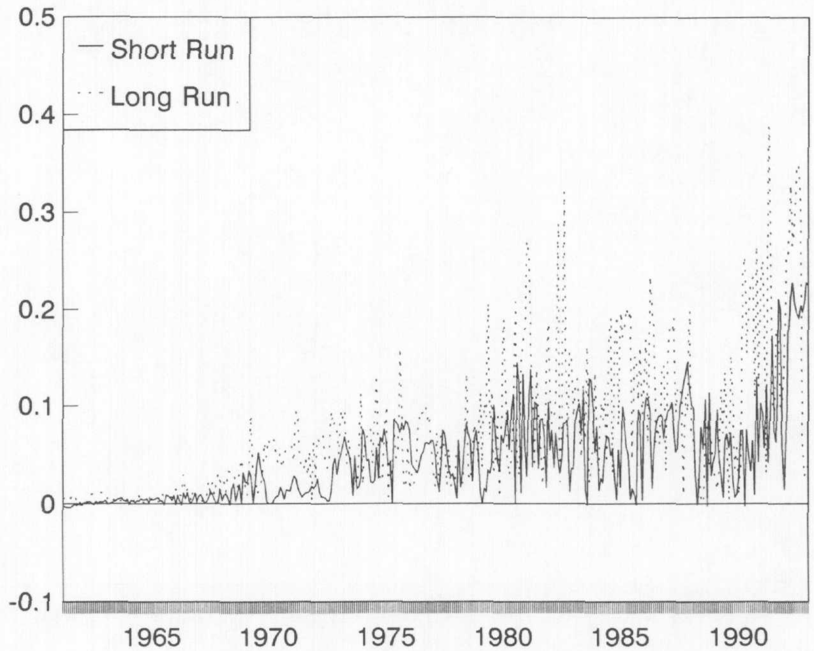

FIG. 2. Elasticities at Eighteen Lags, A1/A3, August 1961–May 1993

trating on short-run elasticities for A2 and A3 over a long run of data.[18] Figure 4 displays the results, this time for a six-month adjustment period for the dynamics. In the figure the solid line represents the McFadden estimates, while the dashed and dotted lines represent the asymmetrical Morishima elasticities. The general conclusion already described clearly holds for this alternative measure. Note, though, that one of the Morishima elasticities (varying the user cost for A2) provides estimates that are very close to the McFadden elasticities, while the other Morishima elasticity does not.

The graphs shown so far are for as many as four hundred monthly observations and, as such, it is hard to see exactly what is going on. The behavior may be revealed in a more interesting way by looking at several recent periods under a microscope. Figure 5 shows the period from January 1980 through December 1982. Here we show the long-run elasticities at eighteen lags for all three monetary assets and we also include, underneath, the user cost of A1, which is the "price" that is being varied in order to produce these elasticities. The recession in 1980 began in January, as suggested by the rise in the user cost, and, not surprisingly, the elasticities jumped somewhat at the same point. More noticeably, the monetary ease that commenced in the Spring of 1980 (and may have produced a sharp fall in the user cost

18. We note that of the many results we looked at, the ones selected for this paper are representative of our findings. We would be pleased to provide other results to the interested reader.

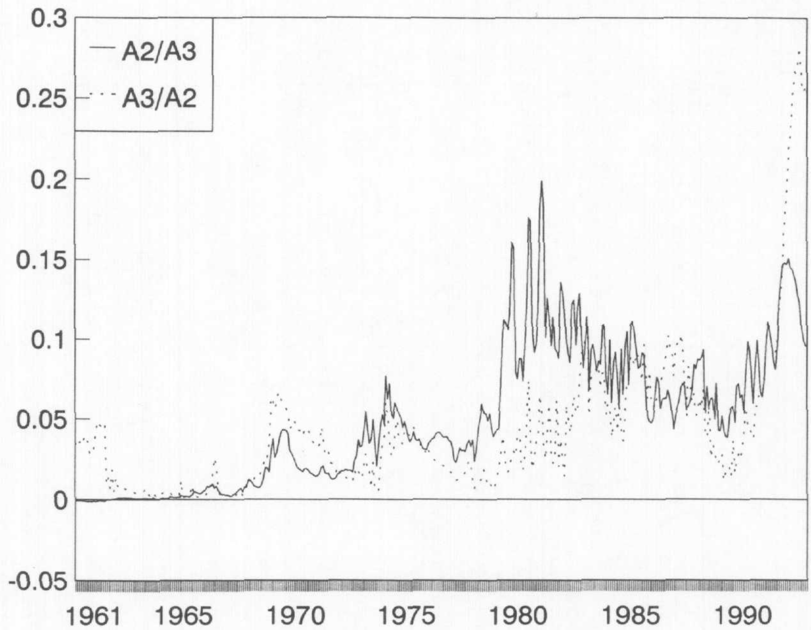

FIG. 3. Short-run Elasticities of Substitution, A2/A3, Six Lags, August 1960–May 1993

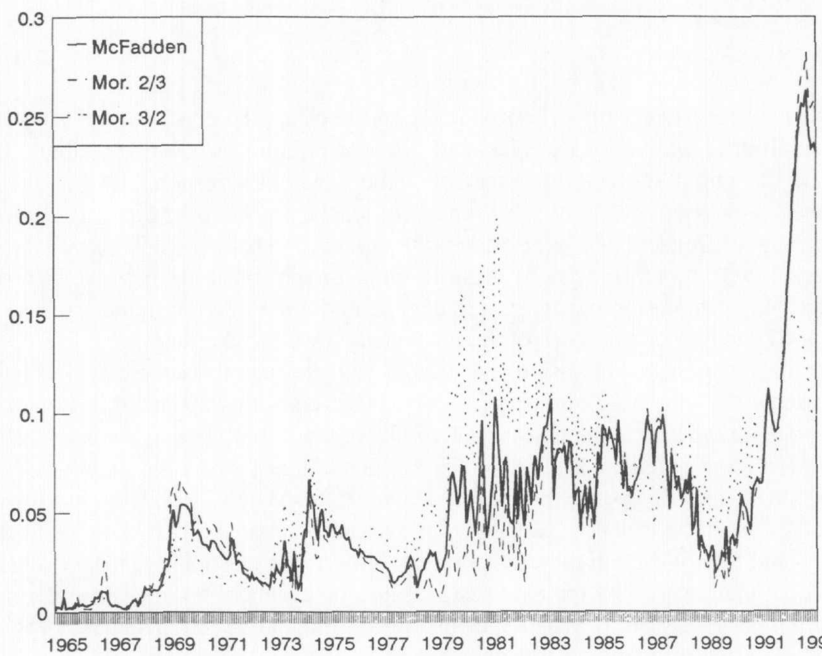

FIG. 4. McFadden versus Morishima Elasticities, January 1965–May 1993

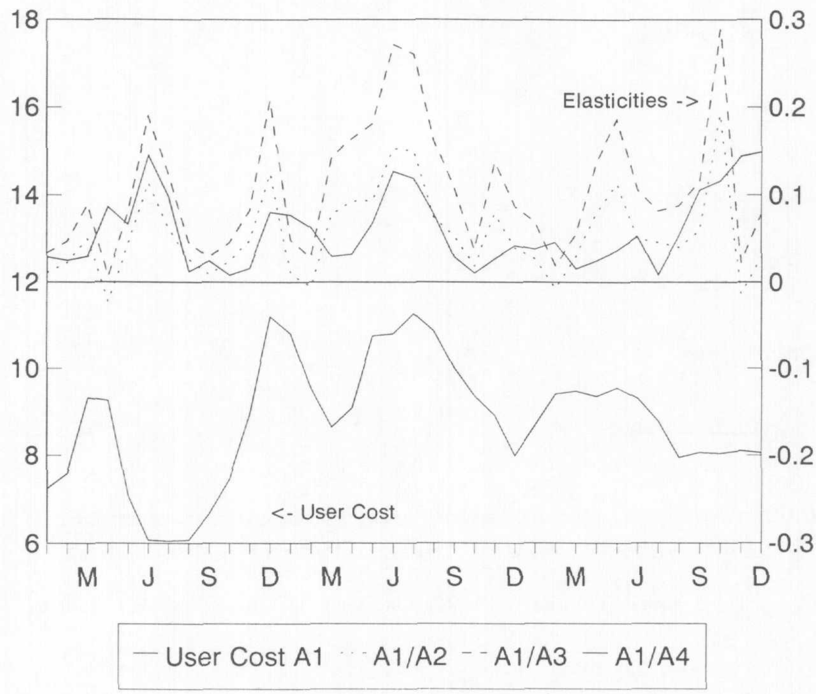

FIG. 5. Long-Run Elasticities at Eighteen Lags, January 1980–December 1982

for A1) is associated with a sharp increase in the estimated elasticities of substitution in all three ways they are calculated. We interpret this as cause and effect. The rest of the graph shows how remarkably the elasticities respond to the sudden changes in the user cost of the most liquid of monetary assets over this period. Such changes were frequent and were often policy induced at this time. We have labeled the graph with monthly dates to make it easier to spot what are now well-known events. We mention the monetary tightness of mid-1981 and the monetary ease of mid-1982, as further examples of the phenomenon we are examining.

We may examine the recession of 1990 and 1991 through the same lens. In Figure 6 we show the same long-run elasticities and user costs that appeared in Figure 5. Putting aside the blip in the first part of 1990, we see a sharp rise in the user cost of A1 and in the elasticities of the financial assets in August 1990, the month that the recession started. Before the recession was over (after March 1991) there is a lowering of the user cost of A1, partly perhaps on account of the easing of monetary policy, and then in March there is more volatility of the elasticities independent, it seems, of the behavior of the user cost. Again, the events of the recession seem to show up as changes of the elasticities of substitution among the financial assets, in the latter case also involving consumer goods.

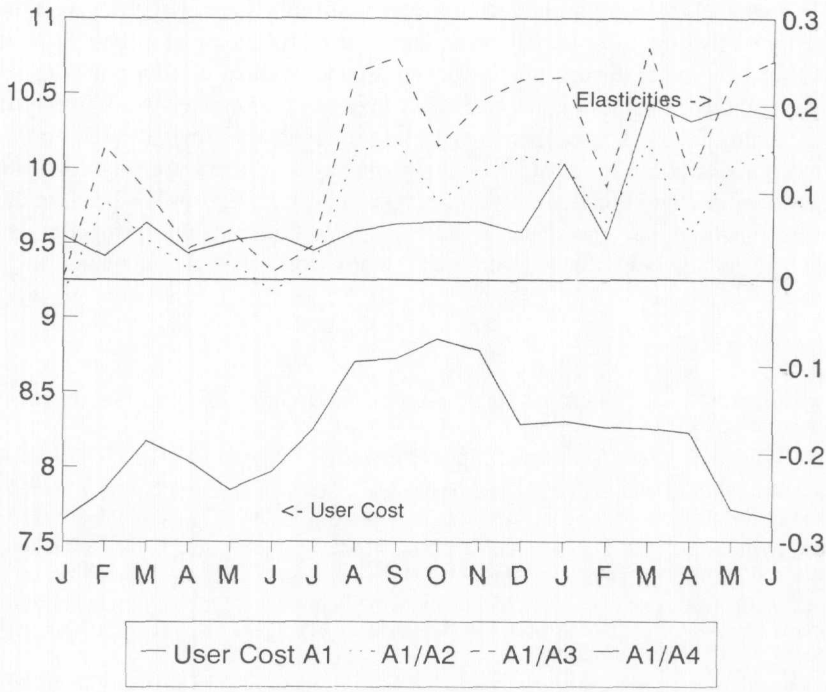

FIG. 6. Long-Run Elasticities, A1/A2/A3, January 1990–June 1991

4. CONCLUSIONS

We think the results of this paper suggest answers to a number of questions raised over previous studies of the demands for financial assets. Most important, we think, is the idea—certainly not original with us—that the traditional measures of money and the traditional log-linear demand for money functions are simply unbelievable in the volatile financial environment in which we find ourselves. The problem, we feel, is that the required assumptions, of constant elasticities of substitution and of close to perfect substitutability among financial assets, simply do not hold. Our findings appear to confirm this.

In this paper we have addressed several other problems that the demand system literature has not simultaneously examined. These include using monthly data, estimating a system of nonlinear dynamic equations, and calculating the asymmetric Morishima elasticities. Our main result is that various financial (and nonfinancial) assets are substitutes in use for each other. In contrast most demand system studies often find complementarity among financial entities using the traditional Allen-Uzawa calculation. The use of Morishima elasticities is the likely source for finding substitution among various financial (and nonfinancial) assets, as the Allen-Uzawa calculation may be incorrect when there are more than two variables.

While we feel these are improvements over the results in the literature, we would like to stress that the most important finding here is still the most important policy conclusion. Financial disturbances, whether around cyclical turning points or not, produce sufficient changes in financial rates of return (as embodied in our user costs) so that wealth-holders are induced to make large enough adjustments in their portfolios to confound the traditional measures of money as well as the traditional log-linear money demand functions. Wealth-holders adjust their portfolios because their response functions, as represented by the aggregate (indirect) utility function estimated here, is highly nonlinear, whether the framework is short run or long run.

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